A systematic study of electronic states in $n-\mathrm{Al}_{x} \mathrm{Ga}_{1-x} A s / G a A s / n-\mathrm{Al}_{x} \mathrm{Ga}_{1-x}$ As selectively doped double-heterojunction structures

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# A systematic study of electronic states in $n-\mathbf{A l}_{x} \mathbf{G a}_{1-x} \mathbf{A s} /$ $\mathbf{G a A s} / \boldsymbol{n}-\mathrm{Al}_{\boldsymbol{x}} \mathrm{Ga}_{1-\boldsymbol{x}} \mathrm{As}$ selectively doped double-heterojunction structures 

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#### Abstract

The electron concentration, the wavefunctions and the energy levels of a $n$ $\mathrm{Al}_{x} \mathrm{Ga}_{1-x} \mathrm{As} / \mathrm{GaAss}^{2}-\mathrm{Al}_{x} \mathrm{Ga}_{1-x} \mathrm{As}^{\text {d }}$ double heterojunction are evaluated by solving Schrödinger and Poisson equations self-consistently. We investigate, at zero temperature, the dependence of the sheet electron concentration, and the subband populations on the well width, spacer thickness and doping concentration, for Al mole fraction $x=0.3$. We give physical interpretations of some interesting characteristics observed. The transition from a 'perfect' square well to a system of 'two separated heterojunctions' is systematically studied. Our results are in excellent agreement with previous experiments.


## 1. Introduction

The quasi two-dimensional electron gas (Q2DEG) which accumulates at the GaAs side of a $n-\mathrm{Al}_{x} \mathrm{Ga}_{1-x} \mathrm{As} / \mathrm{GaAs}$ selectively-doped single heterojunction (SD-SH) exhibits high electron mobilities, especially at low temperatures (Dingle et al 1978, Hiyamizu et al 1983, Hirakawa et al 1986). However, for the sheet electron concentration, $N_{\mathrm{s}}$, which also plays a fundamental role in determining the channel conductivity, a maximum obtainable value has been found to exist, which depends on the structural parameters of the heterojunction (not larger than $1 \times 10^{12} \mathrm{~cm}^{-2}$ ) (Hirakawa et al 1984). An alternative and more effective approach to achieve higher $N_{\mathrm{s}}$ than such a limit, is to use a selectively-doped double heterojunction (SD-DH) (Hamaguchi et al 1984, Inoue et al 1984, Inoue and Sakaki 1984, Inoue et al 1985a, Miyatsuji et al 1985, Burkhard et al 1986, Inoue et al 1986). Studies of rectangular wells formed in an $\mathrm{Al}_{x} \mathrm{Ga}_{1-x} \mathrm{As} / \mathrm{GaAs} / \mathrm{Al}_{x} \mathrm{Ga}_{1-x}$ As structure start to appear in the early seventies (Dingle et al 1974). Although there has been a lot of discussion about the 'inverted interface problem' (Morcoç et al 1981, Sasa et al 1984, Sasa et al 1985), a number of authors have reported that this problem can be overcome in several ways (Drummond et al 1983, Inoue et al 1984, Inoue and Sakaki 1984, Inoue et al 1985b). This was the motivation for the present systematic study of such a system.

Several theoretical methods have been presented in the literature (variational etc), but the most reliable among them are the self-consistent calculations for heterojunctions (Ando 1982a, Ando 1982b, Stern and Das Sarma 1984, Hihara and Hamaguchi 1985, Hurkx and van Haeringen 1985) and SD-DH (Hamaguchi et al 1984, Inoue et al 1984, Inoue and Sakaki 1984, Inoue et al 1985a, Miyatsuji et al 1985, Roan and Chuang 1991). In the case of SD-DH, in recent years, although a number of theoretical studies have been presented, there is a lack of a systematic study, conceming all the parameters involved, bridging the
heterojunctions with the SD-DH structures, and giving a consistent physical interpretation of the results. In the present work we attempt a contribution in this direction. In section 2 we give a description of the SD-DH system, in section 3 the basic theoretical background is presented together with the algorithm used to solve simultaneously, within the effective mass approximation, Schrödinger and Poisson equations. In section 4 the results of the calculations at $T=0 \mathrm{~K}$ are shown and discussed together with comparisons with theoretical and experimental results of other workers.

## 2. Description of the system

In Gigure $1(a)$ a schematic energy diagram of the valence and the conduction band of a $n-\mathrm{Al}_{x} \mathrm{Ga}_{1-x} \mathrm{As} / \mathrm{GaAs} / n-\mathrm{Al}_{x} \mathrm{Ga}_{1-x} \mathrm{As}$ system (the SD-DH structure) is presented for the case in which the materials are considered to be separated. The GaAs well width is $2 L$. The $\mathrm{Al}_{x} \mathrm{Ga}_{1-x}$ As layers, which have finite length, are uniformly doped with donors of concentration $N_{\mathrm{d}}$, except from the two spacer layers of width $D_{s}$, from both sides of the GaAs well. The donor levels are all supposed to lie at an energy $\Delta E_{\mathrm{d}}$ below the bottom of the $\mathrm{Al}_{x} \mathrm{Ga}_{1-x}$ As conduction band. All layers have a small unintentional acceptor doping $N_{\mathrm{a}}$. The acceptor levels are supposed to lie at an energy $\Delta E_{\mathrm{a}}$ above the top of the valence band, although the exact value of $\Delta E_{\mathrm{a}}$ is of no interest here. $\Delta E_{\mathrm{c}}=U_{0}$, and $\Delta E_{\mathrm{v}}$ are the conduction and the valence band offsets, respectively. The layers, considered separated above, are joined to form the heterostructure.
(a)

(b)


Figure 1. Schematic energy diagrams of a SD-DH structure. (a) the materials are considered to be separated. (b) the conduction band of the SD-DH structure, showing the band bending, the various space charge regions, and $n(z)$.

Electrons are moving from the higher lying donor energy levels to the lower lying conduction band and acceptor levels of GaAs. This redistribution of carriers results in the formation of (i) region III which consists of the spacers and the GaAs layer with negative
space charge due to ionized acceptors, (ii) regions II and IV, each of width $D_{d}$, with positive space charge due to ionized donors ( $N_{\mathrm{a}} \ll N_{\mathrm{d}}$ ) and (iii) regions I, V where there is no space charge. The electrons are mainly distributed in the well and their concentration, $n(z)$, which depends only on the coordinate $z$ perpendicular to the interfaces, decays into the barriers, being zero at regions $I$ and $V$. The qualitative diagram of the conduction band is given in figure $1(b)$.

This redistribution of charges causes, together with the potential energy discontinuities and the exchange-correlation potential energy, $U_{\mathrm{xc}}$, what is known as 'band bending'. This is also shown in figure $1(b)$.

The charge density is given by
$\rho(z)= \begin{cases}-\mathrm{e} n(z) & \text { region I: } z \leqslant-\left[D_{\mathrm{d}}+D_{\mathrm{s}}+L\right] \\ \mathrm{e}\left[N_{\mathrm{d}}-N_{\mathrm{a}}-n(z)\right] & \text { region II: }-\left[D_{\mathrm{d}}+D_{\mathrm{s}}+L\right]<z \leqslant-\left[D_{\mathrm{s}}+L\right] \\ -\mathrm{e}\left[N_{\mathrm{a}}+n(z)\right] & \text { region, III: }-\left[D_{\mathrm{s}}+L\right]<z \leqslant\left[D_{\mathrm{s}}+L\right] \\ \mathrm{e}\left[N_{\mathrm{d}}-N_{\mathrm{a}}-n(z)\right] & \text { region IV: }\left[D_{\mathrm{s}}+L\right]<z \leqslant\left[D_{\mathrm{d}}+D_{\mathrm{s}}+L\right] \\ -\mathrm{e} n(z) & \text { region V: } z>\left[D_{\mathrm{d}}+D_{\mathrm{s}}+L\right] .\end{cases}$
For realistic values of $N_{\mathrm{d}}$, all the GaAs layer and spacers are completely depleted. $n(z)$ in regions I, V, as we have already stated, is zero.

The total potential energy is

$$
\begin{equation*}
U(z)=U_{c}(z)+U_{\text {band offset }}+U_{x c}(z) \tag{2.2}
\end{equation*}
$$

whre $U_{\mathrm{c}}(z)$ is the Coulomb potential energy, obtained from the solution of the Poisson equation

$$
\begin{equation*}
\mathrm{d}^{2} U_{\mathrm{c}}(\mathrm{z}) / \mathrm{d} z^{2}=\mathrm{e} \rho(z) / \epsilon_{0} \epsilon \tag{2.3}
\end{equation*}
$$

where, -e is the electron charge, $\epsilon_{0}$ and $\epsilon$, are the vacuum and the relative dielectric constants, respectively. $U_{\text {band offset }}$ is $U_{0}$ for the $\mathrm{Al}_{x} \mathrm{Ga}_{1-x}$ As layers and zero for the GaAs layer i.e. it is the potential energy discontinuity term. The formation of interfaces retains constant the band offsets as long as (i) interfaces are abrupt (ii) band offsets and electron affinities are independent of $N_{d}$ and $N_{a}$, and (iii) no surface charges or dipoles are formed (Heime 1989). For $U_{\mathrm{xc}}$ we adopt the expression given by Hurkx and van Haeringen (1985)

$$
\begin{equation*}
U_{\mathrm{xc}}(z)=-0.0783\left(\mathrm{e}^{2} / \epsilon_{0} \epsilon\right)[n(z)]^{1 / 3} \tag{2.4}
\end{equation*}
$$

which is the dominant term (over 90\%) in the full expression given by Stern and Das Sarma (1984). $U_{\mathrm{xc}}(z)$ is not neglected because it has a small but not negligible influence on the sheet electron concentration (Ando 1982a). According to our calculations, this is 4-5\%. We ignore the image potential energy term (Stern and Das Sarma 1984), because the difference in the dielectric constants of $\mathrm{Al}_{0.3} \mathrm{Ga}_{0.7} \mathrm{As}$ and GaAs is very small. The exact potential energy depends on the specific electron gas distribution. The electron distribution reshapes the well, while it, itself, is reshaped by the shape of the well.

## 3. Theoretical calculations

The above discussion imposes the necessity of simultaneous solution of Schrodinger and Poisson equations. Starting from the Schrödinger equation, applying the effective mass approximation (Slater 1949), taking into account the fact that the $\mathrm{Al}_{x} \mathrm{Ga}_{1-x}$ As effective mass is different from the GaAs effective mass one should use the BenDaniel-Duke equation (BenDaniel and Duke 1966). However, the penetration of the envelope function into the
$\mathrm{Al}_{x} \mathrm{Ga}_{1-x}$ As layers turns out to be small, and consequently we can use the value of GaAs effective mass, $m^{*}$, throughout our system. This assumption results in a Schrodinger-like equation, where variables can be separated.

The electron density at $T=0 \mathrm{~K}$ (postulating $E_{\mathrm{F}} \equiv 0$ ) is

$$
\begin{equation*}
n(z)=-\sum_{i, \text { ocevpied }} \frac{m^{*} E_{i}}{\pi \hbar^{2}}\left|\zeta_{i}(z)\right|^{2} \tag{3.1}
\end{equation*}
$$

where $\zeta_{i}(z)$ is the $z$-axis envelope function with corresponding eigenvalues $E_{i}$.
It is now necessary to determine the asymptotic behaviour of the potential energy $U(z)$ in the bulk material, away from the well i.e. in the regions I and V, and impose the boundary conditions for the envelope functions. At $T=0 \mathrm{~K}, E_{\mathrm{F}}$ can be chosen as the highest populated level. Clearly, this is the donor level $E_{\mathrm{D}}$; thus, $E_{\mathrm{F}}=E_{\mathrm{D}}$. Then, the energy distance between the Fermi level and the bottom of the conduction band is $\Delta E_{\mathrm{d}}$. We have already postulated $E_{\mathrm{F}} \equiv 0$. Therefore the asymptotic values of the potential energy are

$$
\begin{equation*}
U(\text { bulk })=\Delta E_{\mathrm{d}} \tag{3.2}
\end{equation*}
$$

On the other hand, $\zeta_{i}(z)$ decay exponentially in regions I and V.
The algorithm used to solve simultaneously the envelope function equation and the Poisson equation consists of the following steps:
(i) Start with an initial guess of the potential energy $U_{\text {in }}$ (for example the band offsets' potential energy) and solve the one-dimensional Schrödinger-like $z$-axis envelope function equation. In this way, we obtain $E_{i}$ and $\zeta_{i}(z)$.
(ii) Calculate $n(z)$ from (3.1).
(iii) The charge conservation equation gives

$$
\begin{equation*}
D_{\mathrm{d}}=\left[N_{\mathrm{s}}+2\left(L+D_{\mathrm{s}}\right) N_{\mathrm{a}}\right] / 2\left(N_{\mathrm{d}}-N_{\mathrm{a}}\right) \tag{3.3}
\end{equation*}
$$

where the sheet electron density is

$$
\begin{equation*}
N_{\mathrm{s}}=\int_{\text {bulk }(I)}^{\text {bulk }(V)} n(z) \mathrm{d} z \tag{3.4}
\end{equation*}
$$

(iv) Calculate $\rho(z)$ from (2.1).
(v) Obtain $U_{c}(z)$ solving (2.3). In other words integrate $\rho(z)$ twice, starting from distant parts of the left and right sides of the $\mathrm{Al}_{x} \mathrm{Ga}_{1-x}$ As layers (the bulk material), where there is no charge. $D_{\mathrm{d}}$ is usually much smaller than $100 \AA$. In our calculations its largest value does not exceed $600 \AA$, thus, the boundary conditions for the Coulomb potential energy and its derivative are imposed at finite distance from each spacer. At bulk $\mathrm{Al}_{2} \mathrm{Ga}_{1-x} \mathrm{As}$, $U_{\mathrm{xc}}($ bulk $)=0, U$ (bulk) $=\Delta E_{\mathrm{d}}$. Thus,

$$
\begin{equation*}
U_{\mathrm{c}}(\text { bulk })=\Delta E_{\mathrm{d}}-U_{0} . \tag{3.5}
\end{equation*}
$$

Also, in bulk $\mathrm{Al}_{x} \mathrm{Ga}_{1-x} \mathrm{As}$

$$
\begin{equation*}
\left(\mathrm{d} U_{\mathrm{c}} / \mathrm{d} z\right)(\mathrm{bulk})=0 . \tag{3.6}
\end{equation*}
$$

(vi) Calculate $U_{\mathrm{xc}}(z)$ from (2.4).
(vii) Calculate $U_{\text {out }}=U(z)$ from (2.2).
(viii) Is $U_{\text {out }}$ 'very close to' $U_{\text {in }}$ ? If not:
(ix) use Tchebycheff iteration scheme (Akai and Dederichs 1985) to calculate new $U_{\text {in }}$, and so on, until the desired accuracy is succeeded.

## 4. Results

To start with it is necessary to make some remarks concerning the values of the basic parameters involved in our calculations.

The determination of $\Delta E_{\mathrm{c}}$ and $\Delta E_{\mathrm{v}}$ is not a completely solved problem yet (Miller et al 1985, Heime 1989, van Vechten and Malloy 1990, Krijn 1991). However, for Al mole fraction $x=0.3$, which is the case here, the value we use, $\Delta E_{\mathrm{c}}=300 \mathrm{meV}$ is widely adopted (Inoue et al 1984, Hurkx and van Haeringen 1985). Furthermore, we use $\Delta E_{\mathrm{d}}=96 \mathrm{meV}$ which is the experimental value for this Al mole fraction (Ishibashi et al 1982, Watanabe and Maeda 1984). Here it should be noticed that this is a 'mean' value averaging the shallow and $D x$ donors activation energies for this Al mole fraction (Ishikawa et al 1982, Ishibashi et al 1982, Watanabe and Maeda 1984, Watanabe et al 1984, Mizuta et al 1985). The correct value of $\Delta E_{d}$ has not always been used in theoretical calculations, although it is of great importance. For example, Hurkx and van Haeringen (1985), for $x=0.3$, use $\Delta E_{\mathrm{d}}=50 \mathrm{meV}$, instead of the correct value 96 meV ; Cho et al (1987), for $x=0.33$, use $\Delta E_{d}=50 \mathrm{meV}$, instead of the correct value 125 meV . These authors using smaller value of $\Delta E_{\mathrm{d}}$ overestimate the sheet electron concentration. In section 4.1, for both $\mathrm{Al}_{0.3} \mathrm{Ga}_{0.7} \mathrm{As}$ and GaAs , for simplicity, we use the value of $\mathrm{Al}_{0.3} \mathrm{Ga}_{0.7} \mathrm{As} \mathrm{dielectric}$ constant i.e. 12.244 . This results in a slight overestimation of $N_{\mathrm{s}}(0.5-1 \%)$. In the rest of the calculations we use the exact values of the $\mathrm{Al}_{0.3} \mathrm{Ga}_{0.7} \mathrm{As}$ and GaAs dielectric constants i.e. 12.244 and 13.18 , respectively. We obtained these values by interpolating AIAs and GaAs dielectric constants (Jaros 1989). In our calculations, for simplicity we use $m^{*}=0.067 m_{e}$ i.e. its value for GaAs. This value is smaller than the one that should be used taking into account the small penetration of the envelope functions into the $\mathrm{Al}_{0.3} \mathrm{Ga}_{0.7} \mathrm{As}$ layers (Ando 1982a, Ando and Mori 1982). It has been reported (Hiyamizu et al 1983, Mendez et al 1984) that the sheet electron concentration remains constant as a function of temperature up to 100 K . This allows us to compare our theoretical results with experiments performed at 4.2 K and 77 K .

### 4.1. From 'the perfect square well' to 'the double heterojunction'

In figure 2 the sheet electron concentration $N_{\mathrm{s}}$, and the subband populations $N_{\mathrm{i}}$ as a function of the well width $2 L$ are presented. Figure 3 shows $N_{s}$, the subband energies $E_{0}$ to $E_{3}$, and the energy of the centre of the bottom of the well $E_{\text {bot }}$, as a function of the well width $2 L$.

From (3.1) we obtain

$$
\begin{equation*}
N_{\mathrm{s}}=-\sum_{i, \text { occupiod }} \frac{m^{*} E_{i}}{\pi \hbar^{2}} \sum_{i, \text { occupied }} N_{i} \tag{4.1}
\end{equation*}
$$

where $N_{i}$ is the $i$ th occupied subband population. In a 'perfect square well' the well width increase leads to a lowering of the energy levels (Bastard 1988). Consequently, increasing $2 L, N_{s}$ increases. However, when $E_{0}$, the lower subband energy level, touches the bottom of the well a qualitative change begins: the transition from 'the square well' to a system of 'two separated heterojunctions'. As the well becomes wider $E_{0}$ increases, while $E_{1}$ continues to fall, and when both $E_{0}$ and $E_{1}$ are below the bottom of the conduction band in the centre of the well, they can be described as the result of the splitting of the "two heterojunctions' ground states. If the well width continues to increase the 'two heterojunctions' distance increases. The larger is the separation of the 'two heterojunctions' the smaller is the spliting. So, according to our calculations while in a $300 \AA$ well, energy separation between $E_{0}$ and $E_{1}$ is 5.5 meV , in a $600 \AA$ well this energy difference has diminished to 0.03 meV . The mean value of $E_{0}$ and $E_{1}$, which almost exclusively determines $N_{s}$ remains almost constant


Figure 2. The sheet electron concentration $N_{s}$, and the subband populations $N_{1}$ as a function of the well width; $N_{\mathrm{d}}=1 \times 10^{18} \mathrm{~cm}^{-3}, D_{s}=100 \AA$, and $N_{2}=5 \times 10^{14} \mathrm{~cm}^{-3}$.


Figare 3. The sheet electron concentration $N_{\mathrm{B}}$, the subband energies $E_{1}(i=0,1,2,3)$, and $E_{\text {bot }}$ as a function of the well width; $N_{d}=1 \times 10^{18} \mathrm{~cm}^{-3}, D_{2}=100 \AA$, and $N_{\mathrm{a}}=5 \times 10^{14} \mathrm{~cm}^{-3}$.
$\left(1.3 \times 10^{12} \mathrm{~cm}^{-2}\right.$ ) for $2 L>250 \AA$. Consequently $N_{\mathrm{s}}$ saturates, taking twice the value of the corresponding quantity of the single heterojunction. The subband population $N_{2}$ still belongs to the double heterojunction structure. $E_{2}$ and $E_{3}$ can also be seen as the result of the splitting of the first excited states of the two separated heterojunctions. Thus, while for $2 L=250 \AA$ energy separation between $E_{2}$ and $E_{3}$ is 48 meV , in a $600 \AA$ well this energy difference has diminished to 6 meV . For $2 L=700 \AA$, the energy difference between $E_{2}$ and $E_{3}$ decreases much more to 3.6 meV . We can now look at our system as a system of 'two separate heterojunctions'. Figures 2 and 3 give a full description of the above-mentioned transition.

The value of $N_{\mathrm{s}}$ is the same as the one measured and calculated by Inoue et al (1984) and Inoue et al (1985a), for $2 L=300 \AA$. Also the energy levels, as we can conclude from the figure they present, are in agreement with our calculations. A saturation in a one-side doped square well has been presented, without any explanation, by Cho et al (1987). In that case, increasing the well width, a single heterojunction's limit is reached.

Our results are also confirmed by Sasa et al (1985) who measured the $N_{\mathrm{s}}$ and $N_{i}$ dependence of a SD-DH structure on the well width in the range of $60 \AA-1000 \AA$, for Al mole fraction $x=0.28$ and $N_{\mathrm{d}}=1 \times 10^{18} \mathrm{~cm}^{-3}$. Due to unintentional asymmetric Si doping, on only one side of their well is $D_{s}$ different from zero, thas, the ground states of the two 'separated heterojunctions' (the large well-width limit) do not coincide; consequently $N_{0}$ and $N_{1}$, do not coincide either. The saturation effects and the form of the $N_{s}$ and $N_{i}$ dependence observed are similar to our results. In their structure, electrons start to populate the first subband for $2 L>130 \AA$, while in our symmetrically doped structure, for $2 L>150 \AA$. However, they do not give any physical explanation for the observed behaviour or any calculational details as far as their theoretical results are concerned.

### 4.2. Doping dependence

In figure 4 the sheet electron concentration $N_{s}$, together with the subband populations $N_{i}$, and the depletion length $D_{\mathrm{d}}$ are presented as a function of the doping concentration $N_{\mathrm{d}}$, for $2 L=150 \AA N_{\mathrm{a}}=5 \times 10^{14} \mathrm{~cm}^{-3}$ and $D_{\mathrm{s}}=100 \AA$. Electrons start to populate the first excited subband only for $N_{\mathrm{d}}>1 \times 10^{18} \mathrm{~cm}^{-3}$. A characteristic saturation of $N_{\mathrm{s}}$ ( $1.4 \times 10^{12} \mathrm{~cm}^{-2}$ ) is observed. The same behaviour is also observed for the other values of the well width.

From (3.3) it follows

$$
\begin{equation*}
N_{\mathrm{s}}=2 D_{\mathrm{d}}\left(N_{\mathrm{d}}-N_{\mathrm{a}}\right)-2\left(L+D_{\mathrm{s}}\right) N_{\mathrm{a}}=N_{\mathrm{s}}\left(N_{\mathrm{d}}, D_{\mathrm{d}}\right) \tag{4.2}
\end{equation*}
$$

because we keep $L, D_{\mathrm{s}}$ and $N_{\mathrm{a}}$ constants. Thus,

$$
\begin{equation*}
\mathrm{d}\left(N_{\mathrm{s}}\right)=\left(\partial N_{\mathrm{s}} / \partial N_{\mathrm{d}}\right) \mathrm{d}\left(N_{\mathrm{d}}\right)+\left(\partial N_{\mathrm{s}} / \partial D_{\mathrm{d}}\right) \mathrm{d}\left(D_{\mathrm{d}}\right) \tag{4.3}
\end{equation*}
$$

or

$$
\begin{equation*}
\mathrm{d}\left(N_{\mathrm{s}}\right)=2 D_{\mathrm{d}} \mathrm{~d}\left(N_{\mathrm{d}}\right)+2\left(N_{\mathrm{d}}-N_{\mathrm{a}}\right) \mathrm{d}\left(D_{\mathrm{d}}\right) \tag{4.4}
\end{equation*}
$$

Here we have two opposite mechanisms: (i) taking $D_{\mathrm{d}}=$ constant, it follows that $\partial N_{\mathrm{s}} / \partial N_{\mathrm{d}}>0$. (ii) taking $N_{\mathrm{d}}=$ constant it follows that $\partial N_{\mathrm{s}} / \partial D_{\mathrm{d}}>0$. However, an increase of $N_{\mathrm{d}}$ results in a decrease of $D_{\mathrm{d}}$, i.e. if $\mathrm{d}\left(N_{\mathrm{d}}\right)>0$, then $\mathrm{d}\left(D_{\mathrm{d}}\right)<0$. Thus, the first term in the RHS of (4.3) or (4.4) is positive, but decreases in absolute value as $N_{\mathrm{d}}$ increases, while the second term in the RHS of (4.3) or (4.4) is negative and also decreases in absolute value as $N_{\mathrm{d}}$ increases. From our calculations it follows that for small values of $N_{\mathrm{d}}$, the first term is larger than the second one in absolute value, so with increasing $N_{\mathrm{d}}, N_{\mathrm{s}}$ increases.


Figure 4. The sheet electron concentration $N_{5}$, the subband populations $N_{t}$, and the depletion length $D_{\mathrm{d}}$, as a function of the doping concentration $N_{\mathrm{d}}$, for a characteristic value of the well width $2 L=150 \AA ; D_{s}=100 \AA$, and $N_{n}=5 \times 10^{14} \mathrm{~cm}^{-3}$. Electrons populate the first excited subband only for $N_{d}>1 \times 10^{18} \mathrm{~cm}^{-3}$.

However, as we increase $N_{\mathrm{d}}$, the absolute value of the first term falls faster and finally one cancels the other i.e. saturation occurs.

The charge conservation equation is enriched in the case of single heterojunctions, where the acceptors' depletion length must be taken into account, resulting in additional mechanisms. Following the same philosophy the $N_{s}=N_{s}\left(N_{\mathrm{d}}\right)$ behaviour presented by Hurkx and van Haeringen (1985), although they assume an 'infinite' GaAs layer, can be explained. On the other hand, when acceptors are not taken into account, although this is not a realistic assumption for single heterojunctions, following the same lines saturation will occur (Hihara and Hamaguchi 1985). Saturation will occur when the GaAs layer, which of course is not infinite, is all depeleted. These, together with the $N_{s}=N_{s}\left(D_{8}\right)$ behaviour for these systems, will be analytically presented in the near future.

According to our calculations, variation of $N_{\mathrm{a}}$ from $0.1 \times 10^{14} \mathrm{~cm}^{-3}$ to $100 \times 10^{14} \mathrm{~cm}^{-3}$ has a negligible influence on sheet electron concentration.

### 4.3. Spacer dependence

In figure 5 the sheet electron concentration $N_{5}$, together with the subband populations $N_{i}$, and the depletion length $D_{\mathrm{d}}$ are presented, as a function of the spacer thickness $D_{\mathrm{B}}$, for $2 L=300 \AA ; N_{\mathrm{d}}=1 \times 10^{18} \mathrm{~cm}^{-3}$ and $N_{\mathrm{a}}=5 \times 10^{14} \mathrm{~cm}^{-3}$. Electrons start to populate the second excited subband for $D_{s}<25 \AA$. A continuous fall is observed. The same behaviour is also observed for the other values of the well width.

From (3.3) it follows

$$
\begin{equation*}
N_{\mathrm{s}}=2 D_{\mathrm{d}}\left(N_{\mathrm{d}}-N_{\mathrm{a}}\right)-2\left(L+D_{\mathrm{s}}\right) N_{\mathrm{a}}=N_{\mathrm{s}}\left(D_{\mathrm{s}}, D_{\mathrm{d}}\right) \tag{4.5}
\end{equation*}
$$



Figure 5. The sheet electron concentration $N_{\mathrm{s}}$, the subband populations $N_{1}$ and the depletion length $D_{d}$, as a function of the spacer $D_{d}$ for the characteristic value of the well width $2 L=300 \AA, N_{d}=1 \times 10^{18} \mathrm{~cm}^{-3}$, and $N_{\mathrm{a}}=5 \times 10^{14} \mathrm{~cm}^{-3}$. Electrons populate the second excited subband only for $D_{s}<25 \AA$.
because we now keep $L, N_{\mathrm{d}}$, and $N_{\mathrm{a}}$ constants. Thus,

$$
\begin{equation*}
\mathrm{d}\left(N_{\mathrm{s}}\right)=\left(\partial N_{\mathrm{s}} / \partial D_{\mathrm{s}}\right) \mathrm{d}\left(D_{\mathrm{s}}\right)+\left(\partial N_{\mathrm{s}} / \partial D_{\mathrm{d}}\right) \mathrm{d}\left(D_{\mathrm{d}}\right) \tag{4.6}
\end{equation*}
$$

or

$$
\begin{equation*}
\mathrm{d}\left(N_{\mathrm{s}}\right)=-2 N_{\mathrm{a}} \mathrm{~d}\left(D_{\mathrm{s}}\right)+2\left(N_{\mathrm{d}}-N_{\mathrm{a}}\right) \mathrm{d}\left(D_{\mathrm{d}}\right) \tag{4.7}
\end{equation*}
$$

Here we have two collaborating mechanisms: (i) taking $D_{\mathrm{d}}=$ constant, it follows that $\partial N_{\mathrm{s}} / \partial D_{\mathrm{s}}<0$, (ii) taking $D_{\mathrm{s}}=$ constant, it follows that $\partial N_{\mathrm{s}} / \partial D_{\mathrm{d}}>0$. Here, with increasing $D_{\mathrm{s}}, D_{\mathrm{d}}$ decreases; thus, if $\mathrm{d}\left(D_{\mathrm{s}}\right)>0$ then $\mathrm{d}\left(D_{\mathrm{d}}\right)<0$ and in the RHS of (4.6) or (4.7) both terms are negative. That is why we observe a continuous fall.

Experimental results, for $2 L=300 \AA$, have been presented by inoue et al (1984). Their measured values for $N_{s}$ as a function of the spacer width, are in complete agreement with our theoretical results, with the exception of the experimental point $N_{\mathrm{d}}=0.4 \times 10^{18} \mathrm{~cm}^{-3}$ and $D_{\mathrm{s}}=300 \AA$. A similar behaviour has been experimentally observed in single heterojunctions (Hirakawa et al 1984), and theoretically calculated for a one-side doped square well by Cho et al (1987). Explanation for such $N_{s}=N_{5}\left(D_{5}\right)$ behaviour can be interpreted following the arguments used above, taking into account in the case of single heterojunctions the existence of depletion length for acceptors.

## 4.4. $N_{s}$ enhancement

In the structure under study two factors control the magnitude of $N_{s}$. The variation of the doping concentration $N_{\mathrm{d}}$, and that of the spacer thickness $D_{\mathrm{s}} . N_{\mathrm{s}}=N_{\mathrm{s}}\left(N_{\mathrm{d}}, D_{\mathrm{s}}\right)$ is a

2D surface. Starting from any point on this surface, one may move in two perpendicular directions, corresponding to constant $N_{\mathrm{d}}$ or $D_{8}$. In the $N_{\mathrm{d}}$ direction saturation occurs, while in the $D_{\mathrm{s}}$ direction a continuous increase of $N_{\mathrm{s}}$ is observed as we decrease $D_{\mathrm{s}}$ to zero.

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